QP CODE: 26020052

26020052

Reg No : .....

Name : .....

B.Sc DEGREE (CBCS) IMPROVEMENT/ REAPPEARANCE / MERCY CHANCE EXAMINATIONS, APRIL 2026

Fourth Semester

Core Course - MM4CRT01 - VECTOR CALCULUS, THEORY OF NUMBERS AND LAPLACE TRANSFORMS

(Common for B.Sc Computer Applications Model III Triple Main, B.Sc Mathematics Model I, B.Sc Mathematics Model II Computer Science)

2017 Admission Onwards

A2D3EC73

Time: 3 Hours Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Define continuity for a vector-valued function $\mathbf{r}(t)$. Also give an example for a discontinuous vector-valued function.
2. Define the unit tangent vector to a smooth curve. Find the unit tangent vector for the counterclockwise motion $\mathbf{r}(t) = (\cos t)\mathbf{i} + (\sin t)\mathbf{j}$ around the unit circle.
3. Give the equations for the **tangent plane** and the **normal line** at a point on a smooth surface in space.
4. Is $F(x, y) = (3x^2 + 6xy)i + (3x^2 - y^3)j$ a conservative field ? Why/Why not?.
5. Define Curl of a vector field F.
6. State Divergence Theorem.
7. Check whether the following set S of integers constitute a complete set of residues modulo 7 or not:
   $S = \{-12, -4, 11, 13, 22, 82, 91\}$.
8. Verify that $5^{38} \equiv 4 \pmod{11}$ using Fermat's theorem.
9. Check whether the integer 1729 is an $absolute\ pseudoprime$ or not.
10. State Existence theorem for Laplace Transforms.

11. Evaluate $\mathscr{L}(\sin^2 \omega t)$.

12. Using convolution theorem, evaluate $\mathscr{L}^{-1}\left[\frac{1}{s(s-a)}\right]$.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Give the standard parametrization of the line through $P_0(x_0, y_0, z_0)$ parallel to $\mathbf{v} = v_1\mathbf{i} + v_2\mathbf{j} + v_3\mathbf{k}$. Also write two different parametric equations for the line through $P(-3, 2, -3)$ and $Q(1, -1, 4)$.

14. Without finding $\mathbf{T}$ and $\mathbf{N}$, write the acceleration of the motion $\mathbf{r}(t) = (\cos t + t\sin t)\mathbf{i} + (\sin t - t\cos t)\mathbf{j},\ t > 0$in the form $\mathbf{a} = a_{\mathrm{T}}\mathbf{T} + a_{\mathrm{N}}\mathbf{N}$.

15. Find the line integral of $f(x, y, z) = 2x + 3y + z$ over the straight line segment from $(0, 1, -1)$ to $(1, 1, 2)$.

16. Explain and model the equation for work done by a force F over a smooth curve $r(t)$ in space.

17. Find the flux of the field $F = yi - xj + k$ across the portion of the sphere $x^2 + y^2 + z^2 = a^2$ in the first octant in the direction away from the origin.

18. Let a and b are integers that are not divisible by the prime p, then if $a^p \equiv b^p \pmod{p}$ prove that $a^p \equiv b^p \pmod{p^2}$.

19. If$d|n$, then prove that $\phi(d)|\phi(n)$.

20. State and prove first shifting theorem for Laplace Transform.

21. Find $\mathscr{L}^{-1}\left\{\frac{1}{s^2(s^2+\omega^2)}\right\}$.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22.

1. Define the **principal unit normal** vector for a smooth plane curve. Find the principal unit normal vector for the circular motion $\mathbf{r}(t) = (\cos 2t)\mathbf{i} + (\sin 2t)\mathbf{j}$.
2. Find and graph the **osculating circle** of the parabola $y = x^2$ at the origin.

23. State Green's Theorem and apply it to find the counter clockwise circulation and outward flux for the field $F = (x - y)i + (y - x)j$ and the curve C is bounded by the square $x = 0, x = 1, y = 0, y = 1$.

24.

1. State and prove Wilson's theorem.
2. If p is a prime, prove that for any integer a, $p|a^p + (p-1)!a$.

25.

1. Using Laplace Transform, solve
   $y'' + 2y' + 5y = 50\,t - 150,\ y(3) = -4,\ y'(3) = 14.$
2. Solve the Volterra integral equation of the second kind
   $y(t) - \int_0^t y(\tau)\sin(t-\tau)d\tau = t.$

(2×15=30)